Australian
Journal of
Soil Research

volume 39 number 2 2001

Soil Research

39/2

Australian Journal of Soil Research

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Issued six times a year
ISSN 0004-9573

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Australian Journal
of Soil Research

An international journal for the publication
of original research into all
aspects of soil science

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Aust. J. Soil Res., 2001, 39, 219–238

A comparison using the caesium-137 technique of the relative importance of cultivation and overland flow on soil erosion in a steep semi-tropical sub-catchment

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Abstract

The spatial pattern of net soil loss on 6 downslope transects in a small semi-tropical sub-catchment was measured in 1990–91 using the resident caesium-137 deficit technique. The sub-catchment consisted of 2 opposing hillslopes which shed water to an intermittent stream in the valley bottom of the sub-catchment. There were 3 transects on each of the opposing hillslopes, and measurement indicated net soil loss from all 6 transects. Furthermore, the spatial pattern of caesium-137 deficit did not indicate the accumulation of soil expected due to the slope decrease toward the bottom of the valley. Possible explanations of this finding could be the effect of periodic flooding of the intermittent valley stream, or seepage-accelerated erosion.

Pineapple cultivation in the sub-catchment since 1950 included intensive cultivation at 4-year intervals by downslope-moving rotary hoe. The paper develops a theoretical prediction of the spatial pattern of net soil loss expected due to such cultivation, as well as the expected pattern of soil loss due to overland flow on the hillslopes. The spatial patterns of soil loss due to these 2 different soil erosion mechanisms were then compared with the pattern of net soil loss indicated by caesium-137 depletion to provide an assessment of their likely relative importance in contributing to soil loss. In the upper part of each hillslope, this comparison of spatial trends did not allow the dominant cause of soil erosion to be distinguished. Both the model of erosion due to cultivation and that due to hillside overland flow predicted soil accumulation in the lower valley sides where slope decreased. Neither model represented the net loss of such accumulated soil indicated by caesium-137 deficit, and this loss possibly occurred during periodically observed flooding of the valley floor, or due to surface burial with caesium-137 depleted subsoil.

Additional keywords: soil erosion mechanisms, erosion model, rotary hoe cultivation, pineapple plantation.

Introduction

Soil erosion is a major environmental concern in agricultural practice, especially on steeply sloping land. In addition to erosion by rainfall and overland flow it is increasingly being recognised that cultivation itself can be an important cause of downslope soil displacement, and thus of net soil erosion (Govers *et al.* 1994; Lobb *et al.* 1995). How the relative importance of cultivation and overland flow on soil erosion varies with landscape and cultivation methods is not yet completely understood. However, a recent issue of *Soil and Tillage* with 12 papers devoted to this topic (Govers 1999) indicates a rapidly growing research base on these issues.

Lobb *et al.* (1995) studied erosion on landscapes of variable slope due to tillage operations using a mouldboard plough, tandem disc, and C-tine cultivator, with up- and downslope cultivation. Soil translocated by downslope tillage was more than 1.5 that due to tillage upslope. Using the caesium-137 technique, they also found that tillage erosion accounted for about 70% of indicated total soil loss.

© CSIRO 2001 10.1071/SR99007 0004-9573/01/020219

Govers *et al.* (1994) stated that the erosion associated with tillage operations may be more important than water erosion especially on the hilly agricultural land of Western Europe. The study found that erosion rates associated with tillage may often be more than 10 t/ha.year.

In this paper, the expected spatial pattern of net soil redistribution by cultivation and by overland flow is obtained using physically based models outlined in the paper. Development of the model of soil erosion by cultivation is based solely on the downslope displacement of soil due to rotary hoe cultivation on land of varying steepness. The overland flow model used is based on the theory of soil erosion due to overland flow developed by Hairsine and Rose (1992). Soil samples for caesium-137 analysis were taken and analysed by the authors. Comparison of the expected spatial pattern of net soil redistribution given by these models with the pattern in caesium-137 data is used to assess the relative importance of cultivation and overland flow on soil redistribution on the 2 opposing hillsides of a V-shaped sub-catchment. Use of caesium-137 concentration as an independent reference indicator of net soil redistribution is widely used to represent net soil redistribution (Sutherland and de Jong 1990; Walling and Quine 1991).

Net soil redistribution models

The symbols and abbreviations used in this paper are listed in Appendix 1.

Net soil redistribution by rotary hoe cultivation

Modelling soil redistribution or translocation due to tillage, as far as is known to the authors, has not included cultivation by rotary hoe. The only experimental data available to evaluate the magnitude of net soil erosion in the reported study was that provided by the caesium-137 technique. Cultivation by rotary hoe would incorporate caesium-137 throughout the cultivation layer, probably somewhat uniformly, especially if cultivation is frequent. There appear to be at least 3 ways in which caesium-137 concentration in the cultivation layer can be affected on hillside cultivated by downslope movement of a rotary hoe cultivator. These 3 ways are as follows:

(*i*) There are what might be termed 'start up' effects of cultivation. For a certain distance downslope from the commencement of cultivation, soil removed by the rotary hoe is not recompensed by soil delivered from upslope, as occurs over most of a cultivated hillside. This applies only for a short distance from cultivation commencement. Let us assume that repeated cultivation always commences at the same place on the landscape. Then even recognising that caesium-137 delivery to the soil surface has been over a period of time, it is quite likely that excessive soil removal in this short band of 'start-up cultivation' will eat into subsoil unlabelled by caesium-137. Such unlabelled soil as is removed from this start-up band by the rotary cultivator will then be moved downslope in each successive cultivation event. In a situation of frequent cultivation this downslope dilution of caesium-137 concentration by unlabelled soil will lead to a deficit in caesium-137 concentration, which will be interpreted as greater soil erosion than has actually occurred. This will be called a 'start-up dilution effect'.

(*ii*) There are many sites at which substantial soil erosion (by any mechanism) has occurred in the period where the atmospheric supply of caesium-137 has significantly declined due to modification or cessation of atomic bomb testing. At such sites it is possible that cultivation has extensively incorporated soil from deeper layers unlabelled by caesium-137. If so, then again the normally used caesium-137 method of assessing net soil loss could lead to overestimation of soil loss.

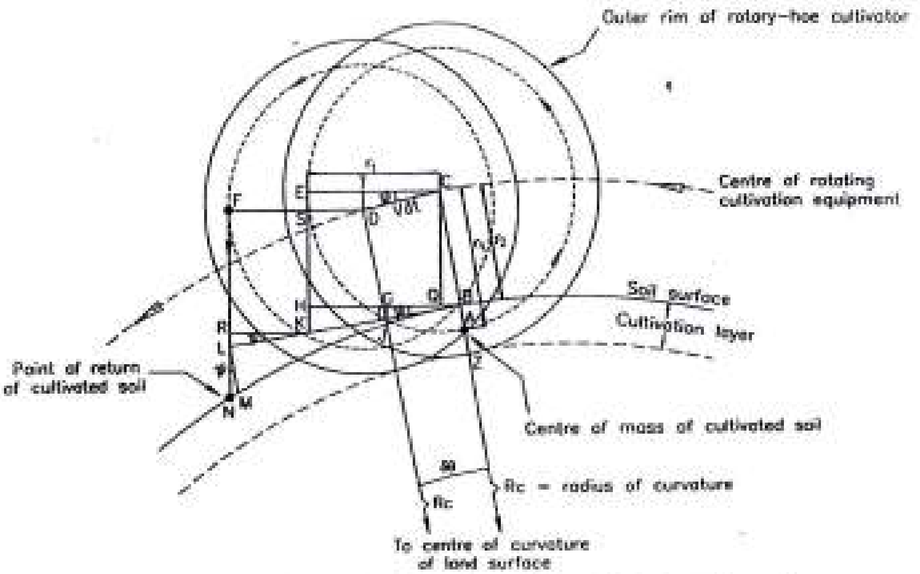

Fig. 1. Movement of rotary hoe in time interval δt on land of non-uniform slope.

The theory given in this paper for net soil loss by cultivation neglects the possible influence of cultivation on caesium-137 concentration described earlier in (*i*) and (*ii*). The only cultivation effect for which theory is developed is that given in (*iii*).

(*iii*) Cultivation can also effect net soil loss, and thus caesium-137 concentration, due to its interaction with change of land surface slope. For land of spatially variable slope, the distribution of soil during downslope cultivation by rotary hoe will be shown to be non-uniform. Figure 1 illustrates the redistribution of soil due to the movement of a rotary hoe cultivator whose centre of rotation has moved from point C to D. If the land was of uniform slope, the soil whose centre of mass is initially of A (Fig. 1) would be moved a downslope distance corresponding to that from B to L. In the context of a uniform slope, such as BL, there would be net loss of soil from B to L, a distance of the order the radius of the rotary hoe, about 1 m. For further movement of the rotary hoe down a uniform slope, soil displaced downslope at any point is replaced by soil from further upslope. Hence, this uniform slope process is commonly interpreted as a small downslope displacement of the entire soil surface rather than a net downslope loss of soil at any point in the landscape. However, in a situation where there is the change of slope, as indicated in Fig. 1, the soil would be removed from B to N rather than from B to L. Therefore, there would be a net downslope local displacement of soil effectively from M to N, where LM is perpendicular to BL (the land slope at B). Theory is developed for this net downslope displacement. The derivation of the distance MN is presented in Appendix 2, where Eqn A15 shows that in time interval δt, the distance:

$$MN = \frac{\left\{ \dfrac{r_1(1+\sin\psi)}{\cos\psi} + v\,\delta t \right\} \tan\psi}{2\cos\psi} \times \frac{\delta S_t}{(1+S_t^{\,2})^{1.5}} \qquad (1)$$

where v is the speed of the rotary hoe's general motion, $S_t = \tan\psi$, r_1 and ψ being shown in Fig. 1.

The distance MN illustrates graphically the effect of rotary cultivation on the net soil redistribution with varying slope. The expression of net soil redistribution due to rotary cultivation is also dependent upon the magnitude of BL, and $\delta\dot{o}$ on v, δt, r_1, and ψ as follows from Appendix 1 (Eqns A7 and A8). Neglecting vδt as small compared with the term involving r_1 in Eqn 1, and neglecting the factor 2 as a constant, the net soil redistribution is given in the following proportionality equation:

$$\text{Net soil redistribution} \propto \frac{(1+\sin\psi)\tan\psi}{\cos^2\psi\,(1+St^2)^{1.5}} \times \delta St \qquad (2)$$

Also, the net redistribution or cultivation erosion factor of soil due to cultivation will be defined as MN/δs where δs is the arc length BN. Now regarding δs as a small quantity, δs = δx/cos ψ where x is distance on a horizontal axis. Therefore, we define MN/δs, the cultivation erosion factor, as follows:

$$\text{Cultivation erosion factor} \propto \frac{MN}{\delta s} \propto \frac{(1+\sin\psi)\tan\psi}{\cos\psi(1+St^2)^{1.5}} \times \frac{\delta St}{\delta x} \qquad (3)$$

If y = f(x) is a description of the land surface geometry, then S_t = tan ψ = dy/dx. Thus in the limit, δSt/δx in Eqn 3 is equal to $d^2f(x)/dx^2$. Therefore, the cultivation erosion factor (CEF) will be defined as:

$$CEF = \left[\frac{(1+\sin\psi)\tan\psi}{\cos\psi\,(1+St^2)^{1.5}}\right] \times \frac{d^2f(x)}{dx^2} \qquad (4)$$

The magnitude of the numerator of the CEF, (1 + sin ψ) tan ψ, will increase with the angle ψ, as does the denominator, cos ψ $(1 + St^2)^{1.5}$. Thus, since these 2 components in Eqn 4 tend to approximately oppose one another, the dominant term in the cultivation factor will be $d^2f(x)/dx^2$.

The spatial variation in net soil redistribution can be determined graphically using the CEF (Eqn 4), which is calculated using the microtopographic data of the sub-catchment. A positive value of the cultivation factor implies that net erosion has occurred, whilst a negative value indicates net accumulation of soil.

Net soil redistribution due to overland flow

The expectation of net soil redistribution due to overland flow is based upon the flow of water and sediment down a sloping land surface without inflow to the top of the hillslope. The generalised representation of downslope erosion will be made without specific representation of the role of rills or of net downslope rainfall detachment. The presence of rills does not invalidate the method of erosion representation employed, but would affect the value of a parameter k in Eqn 12 given later, adding some 'noise' to the water erosion factor of Eqn 13. The rates of water and sediment flowing per unit time across a unit width of the plane are given as volumetric water flux, q, and sediment flux, q_s, respectively. The average velocity in a uniformly turbulent overland flow is given by Manning's equation as follows:

$$V = D^{2/3}\,\frac{S^{1/2}}{n} \qquad \text{where } q = DV$$

and where S is land slope (sine of slope angle), V is the velocity of water flow (m/s), D is depth of flow (m), and n is the surface roughness coefficient called Manning's n ($m^{-1/3}$ s). Using Manning's equation for the sheet flow, the volumetric water flux, q (m^3/m.s), can be expressed as:

$$q = \frac{n^{3/2}\, V^{5/2}}{S^{3/4}} \tag{5}$$

The volumetric water flux, q, can also be expressed in terms of runoff rate per unit area, Q, and downslope distance, s, measured from the ridge crest. Using the approximation of Rose *et al.* (1983), q is given by:

$$Q = Qs$$

Hence the value of V in Eqn 5 can be expressed as:

$$V = \frac{S^{3/10}}{n^{3/5}}(Qs)^{2/5} \tag{6}$$

In order to estimate soil loss from the plane, the sediment flux, q_s, is required. The value of q_s can be inferred from the sediment concentration and runoff collected during a rainfall event. Direct measurement of net sediment flux over the surface at a sub-catchment scale is difficult. However, the spatial form of soil loss can be represented using a simplified version of a model of soil erosion which has received validation elsewhere.

It is recognised that there is an upper limit to sediment concentration, commonly called the transport limit (Foster 1982). Simplifying the expression for sediment concentration at the transport limit, c_t (kg/m^3), given by Hairsine and Rose (1992) shows that c_t increases with land slope, S, and flow velocity, V. The term c_t is then given by:

$$c_t = KSV \tag{7}$$

where K is a constant for any soil, but may vary with soil characteristics. Substituting Eqn 6 into Eqn 7, then:

$$c_t = K\frac{S^{1.3}}{n^{0.6}}(Qs)^{0.4} \quad \text{or}$$

$$c_t = K_1 S^{1.3} s^{0.4} \tag{8}$$

where

$$K_1 = \frac{KQ^{0.4}}{n^{0.6}}$$

Rose (1993) represented the relationship between the sediment concentration, c (kg/m^3), and c_t by a soil erodibility parameter, β, such that:

$$c = c_t \beta$$

where the value of β is expected in general to be ≤1. For areas of hillslope seepage it is possible that β might significantly exceed 1, possibly being up to 1.5 (Huang *et al.* 1998). Thus the sediment flux, q_s (kg/m.s), is:

$$q_s = qc = Qsc_t^{\beta}$$

Using Eqn 8 for c_t, then:

$$q_s = Qs\,(K_1 S^{1.3} s^{0.4})^{\beta} \qquad \text{and so}$$

$$q_s \propto S^{1.3\beta}\, s^{(1+0.4\beta)}$$

$$q_s = k\, S^{1.3\beta}\, s^{(1+0.4\beta)}$$

Because the value of β is somewhat uncertain, and may vary with distance downslope, the further simplifying approximation will be made that:

$$q_s(s) = k\, S^a\, s^b \tag{9}$$

where k is approximately constant. Increasing the downslope distance from s to s+δs leads to the sediment flux being given by:

$$\begin{aligned} q_s(s+\delta s) &= k(S + \frac{dS}{ds}\delta s)^a (s+\delta s)^b \\ &= k\,S^a(1 + \frac{a}{S^a}\frac{dS}{ds}\delta s)\, s^b\,(1 + \frac{b\,\delta s}{s^b}) \\ &= k\,[S^a s^b + S^a b\,\delta s + a\frac{dS}{ds} s^b\,\delta s + a\,b\frac{dS}{ds}(\delta s)^2\,] \end{aligned}$$

Neglecting the second-order term, then:

$$q_s(s+\delta s) = k\,[S^a s^b + S^a b\,\delta s + a\frac{dS}{ds} s^b\,\delta s] \tag{10}$$

The difference between $q_s(s+\delta s)$ and $q_s(s)$ gives the net movement of sediment flux, or soil redistribution per unit width. The net soil redistribution per unit width, $\Delta\, q_s(s)$, at a downslope distance, s, is then given by:

$$\Delta q_s(s) = k\,\delta s\left[S^a\, b + a\, s^b\,\frac{dS}{ds}\right] \tag{11}$$

Therefore, the net soil redistribution per unit area, NE(s), is given by dividing Eqn 11 by δs, to give:

$$NE(s) = k\left[S^a\, b + a\, s^b\,\frac{dS}{ds}\right] \tag{12}$$

The magnitude of net soil redistribution per unit area, NE(s), is a function of slope (S), downslope distance (s), and the spatial rate of change of S with s (i.e. dS/ds). Using Eqn 12, the dependence of net soil redistribution due to overland flow of water is represented by a water erosion factor (WEF) as follows:

$$WEF = S^a b + a\, s^b\,\frac{dS}{ds} \tag{13}$$

The value of a and b is determined by the value of β which, based on previous experience, is likely to be in the following ranges:

(*i*) for top and midslope: $\beta = 0.77 \rightarrow 1.0$; and (*ii*) for bottomslope: $\beta = 1.0 \rightarrow 1.5$.
Since $a = 1.3\beta$, and $b = (1 + 0.4\beta)$, the value of WEF was explored for the range of values of a and b given in Table 1.

Table 1. The range of values of the parameters 'a' and 'b' explored due to uncertainty in the erodibility parameter β

Slope portion	Value of 'a'	Value of 'b'
Top and mid-slope	$1 \rightarrow 1.3$	$1.3 \rightarrow 1.4$
Bottom slope	$1.3 \rightarrow 1.95$	$1.4 \rightarrow 1.6$

Eqn 13 shows that both the magnitude of slope (S) and rate of slope change (dS/ds) at each part of the transect determine whether net erosion or deposition occurs in the overland flow. A negative value of WEF implies that net accumulation is taking place, but a positive value implies net erosion. If the slope is convex, then dS/ds is positive so that net erosion or net soil loss must occur. However, when the slope is concave, so that dS/ds is negative, then net deposition can take place, depending on the relative magnitude of the 2 terms in Eqn 13.

Methods

Experimental site

The study was undertaken on a sub-catchment of a privately owned pineapple farm at Imbil, about 170 km north of Brisbane, which is cultivated for pineapples. The sub-catchment is characterised by steeply sloping land, the mean slope being approximately 29%, but slope varied in the experimental area between 19% and 37%. The soil type of the sub-catchment is a Lithic Eutropept or a clayey sandy breccia (using the nomenclature of Piper and Rogers 1980, cited in Collinson and Thompson 1982). The soil can be described as gravelly, mildly acidic, weak-crumbed loam to clay loam with an A_1 horizon of 5–30 cm, with gravelly bleached A_2 horizon. The sub-catchment lies within the humid sub-tropical region with a summer-dominant rainfall. The average annual rainfall was 1350 mm with a maximum average monthly rainfall occurring in January (182 mm), and a minimum in August (38 mm). More details of the Imbil site are given by Ciesiolka *et al.* (1995).

The sub-catchment study area is about 3.5 ha, divided into 2 steep, cultivated hillslopes opposing each other from which runoff was collected by a centrally located, intermittently flowing creek. These 2 opposing hillslopes will be referred to as the north and south sections. The sub-catchment was divided into 18 plots (9 plots on each section), the plot boundaries being approximately normal to contour lines. Transect 1 is towards the top of the catchment and transect 9 further down the sub-catchment for both the north and south hillslopes. Pineapples were planted in rows up-and-down the local slope within these plots (i.e. in the direction of steepest descent) for a 4-year crop cycle, then cleared and cultivated again for another cropping cycle. This pattern of pineapple cultivation had been practised since 1950. On completion of a crop, intensive cultivation is undertaken on this steeply sloping land using a rotary hoe cultivator in order to kill the previous pineapple crop, incorporate plant material into the soil, and provide suitable soil conditions for the next planting. The rotary hoe is operated when travelling downslope, with cultivation commencing near the top of the hillslope denoted as the origin of downslope distance in Figs 2–4.

Sampling techniques for resident caesium-137

All caesium-137 samples from the cultivated area and a nearby reference site (selected as most unlikely to have experienced soil erosion or deposition) were collected in late 1990 to early 1991. The reference site was on a flat ridge of uncultivated bushland located several hundred metres above the head of the experimental sub-catchment. Two samples were obtained from this reference site.

At the cultivated site, 3 plots within the north section (plots 1N, 5N, and 9N) and 3 plots in the south section (1S, 5S, and 9S) were chosen as sampling sites. Three sampling points along a transect were chosen within each plot, being taken from the top (T), middle (M), and bottom (B) of the transect. This sampling provided 3 replicate profiles from each of the opposing north and south facing hillslopes. The top sample (T) was taken sufficiently downslope of the commencement of rotary hoe cultivation to avoid the initial startup net soil loss that would occur even on a uniform slope. However, this top sample could be affected by the startup dilution effect described earlier in the 'model' section.

The soil samples at the reference site were collected using a scraper-plate within a rectangular frame (20 by 20 cm) sampling soil from the 0–5 cm, 5–10 cm, and 10–15 cm depth increments, respectively. The area planted to pineapples was cultivated by a rotary hoe to a depth of 15 cm many times at the end of each 4-year cropping cycle (sometimes >10 cultivations being needed to prevent re-sprouting from leaf parts). There were 9 or 10 cropping cycles since the first atmospheric distribution of caesium-137 in 1952 until soil sampling in 1990–91. Thus, there could have been about 100 rotary hoe cultivations during the period of caesium-137 fallout which would have rather thoroughly incorporated the caesium-137 through the 15-cm depth of cultivation. This was assumed to justify taking only 0–5 cm samples from the cultivation sites for caesium-137 determination, using the same sampling method previously described for the reference site; the areal caesium-137 concentration was then multiplied by the ratio 15/5, or a factor of 3, to make a more valid comparison with measurements from the reference site where sampling was to a depth of 15 cm.

Caesium-137 analysis

All samples from both the reference and cultivated sites were prepared for caesium-137 analysis following the guidelines given by Walling and Quine (1991). Caesium-137 activity per unit mass (Bq/kg) was determined by counting the gamma emissions at 662 keV with a germanium semi-conductor detector (McCallan *et al.* 1980). Areal activity of caesium-137 (Bq/cm^2) of each sampling point was calculated using the described method by Kachanoski (1987), Martz and de Jong (1987), and Sutherland and de Jong (1990).

Caesium-137 accumulation or loss at each sampling point can be determined by subtracting the average caesium areal activity at the reference site from the areal activity at each sampling point following the method of Sutherland and de Jong (1990). When the caesium areal activity at the sampling point was higher than at the reference site, soil accumulation was estimated to occur, otherwise soil loss was estimated to take place. The caesium-137 accumulation or deposition can be used to determine net soil accumulation or loss at each sampling point. The net soil accumulation or loss in this study was calculated using the proportional method as outlined in Sutherland and de Jong (1990).

Results and discussion

Topography

The topographic data were obtained using standard survey techniques. Land slope varies with downslope distance but in a manner specific to each transect taken normal to contour lines. The lowest slopes were found at the crest of the hillslope of each transect. In general, slope increased sharply with downslope distance, and then decreased slightly toward the bottomslope after reaching a maximum slope in the middle part of the transect. The only transects with a trend different to this were transects 9N and 5N, where the slope increased from the top to the middle area of the transect, and then varied slightly until reaching the highest slope at or close to the bottom of the transect (see Fig. 2). Figure 2 also includes the sine of the slope angle, ψ, involved in the CEF (Eqn 4). Downslope distance was measured from the commencement of rotary hoe cultivation.

Caesium-137 results

The caesium-137 analysis showed caesium-137 activity per unit area at all sampling points of the cultivated site to be significantly lower than those at the reference site. If the assumptions made are correct then this implies that net soil loss had occurred in all transects prior to sampling. The location and calculated magnitude of these losses in t/ha.year for each of the 3 sampling positions in each transect is given in Fig. 2.

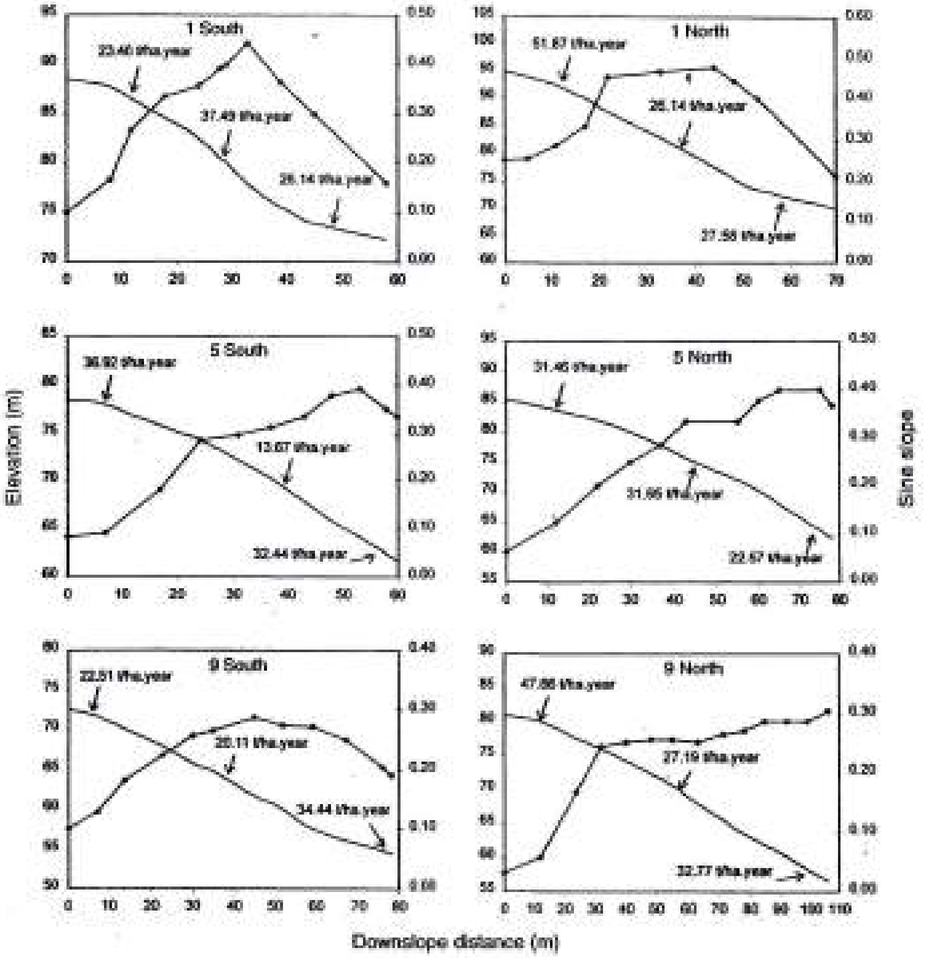

Fig. 2. Microtopographic feature and net soil loss profile for every transect. Slope is defined as the sine of the surface slope angle, and net soil loss at the three sample sites in each transect based on caesium measurements is given in units of t/ha.year. — Elevation; –O– sine slope.

Net erosion was expected at the top and midslope sampling position in agreement with measurement (Fig. 2). However, net deposition or accumulation of sediment was expected for those lower sampling sites where slope was decreasing (which was the case in all but 9N, as shown in Fig. 2). There are at least 3 possible reasons for this unexpected indication of soil loss at the lower sampling sites. Firstly, periodic flooding of the valley floor is known to occur, and the position of the lower sampling sites in the landscape is such that even if periodic accumulation of caesium-137 labelled sediment did occur, it could have been removed by floodwaters prior to sampling for caesium-137 measurement.

A second reason for the measured deficit in caesium-137 at lower sampling sites would be that valley flooding led to net deposition at these sites of soil unlabelled with caesium-137, originating perhaps from stream banks or incised rills. If this was the case sampling may not have been deep enough to capture all the caesium-137 possibly buried deeper in the soil profile.

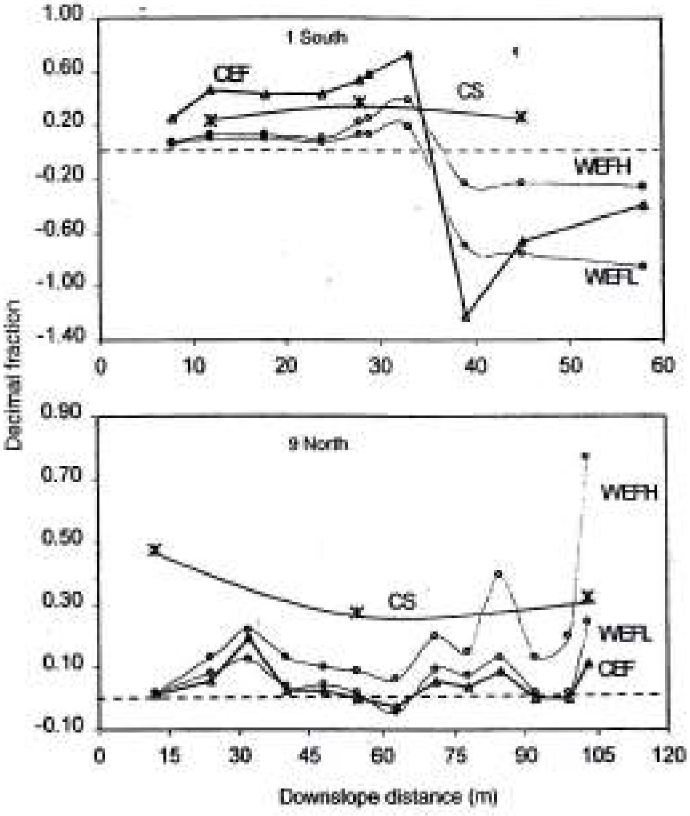

Fig. 3. Comparison of trend of the net soil loss (t/ha.year$\times10^2$) based on caesium-137 (CS) with the cultivation erosion factor (CEF) and the water erosion factor calculated used high (WEFH) and low (WEFL) values of 'a' and 'b' from Table 1.

A third possible explanation is that erosion at this lower site was exaggerated by the positive pore water pressures caused by the exfiltration of seepage. Huang *et al.* (1998) have shown that this little-appreciated effect of hillslope subsurface hydrology can increase erosion rates well above what otherwise would be expected. This would imply a value of b substantially greater than unity in the theory of overland flow driven erosion, a possibility acknowledged in the values of a and b for the bottomslope in Table 1.

The data on net soil loss within the sub-catchment given for each sample site in Fig. 2 show that, on average, the highest net soil loss occurs on the upper slope section of the transects. McFarlane *et al.* (1992), and Elliott and Cole-Clark (1993), who studied soil loss on potato farms at Western Australia and NSW, respectively, found similar results in that net soil erosion was most severe on the upper slopes. The mean rate of net soil loss on the upper slope, midslope, and bottomslope is 35.6, 26.0, and 29.3 t/ha.year, respectively. It is possible that the indicated mean soil loss of 35.6 t/ha.year for the top upper slope sample is an overestimate because of the start-up dilution effect with unlabelled soil which was explained in the model section. However, based on the measured caesium-137 deficit, the second highest net soil loss is from the bottom of the transect, whereas net deposition at this

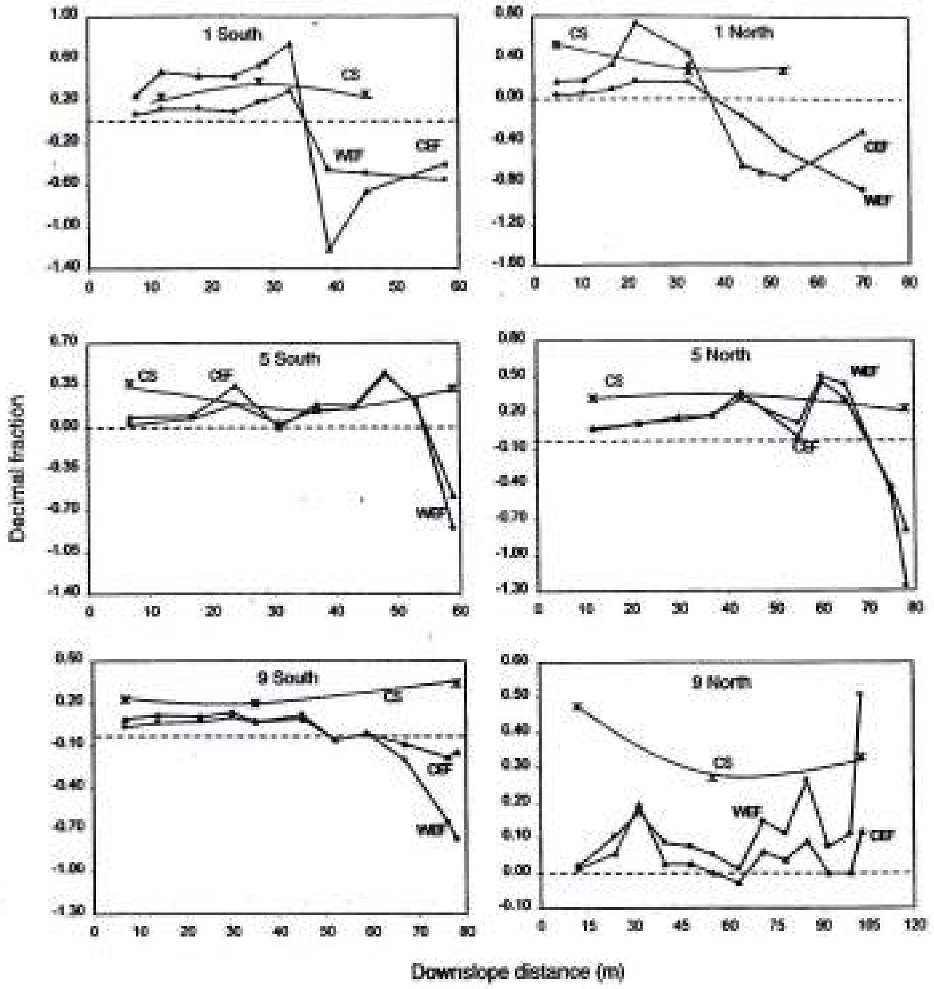

Fig. 4. Comparison of trend of the net soil loss (t/ha.year×10^2) based on caesium-137 (CS), mean water erosion factor (WEF) and cultivation erosion factor (CEF) for all transects .

site was expected on topographic grounds, a finding for which earlier discussion developed 3 possible explanations.

Water erosion factor

The trend of net soil redistribution by overland flowing water was determined by calculating WEF in Eqn 13 using the appropriate range of values of a and b given in Table 1. As calculation progressed downslope, a gradual transition in values from top and midslope to bottomslope was used in the calculations. Calculations were carried out using the upper limit to values chosen for 'a' and 'b', and then the lower limit (Table 1). The outcome of such calculations is illustrated for 2 transects in Fig. 3, which gives 2 estimates, WEFH for high, and WEFL for low values of 'a' and 'b'.

The results in Fig. 3 typify the outcome for all transects, showing that the general trend was the same for both high and low estimates, the trend beigg exaggerated in the case of WEFH. With this justification, and for greater simplicity of presentation, only the trend of WEF using mean values of 'a' and 'b' is given for all transects in Fig. 4.

Net soil loss due to the overland flow, characterised by a positive value of the WEF, occurs over at least the first 40 m of all transects, and with the exception of transect 9N, WEF becomes negative towards the bottom of the transect, indicating net soil accumulation. For transect 9N, net erosion by flowing water is indicated for the entire transect.

Comparing Figs 2 and 4 it can be seen that over most transects, the trend of net soil redistribution within the sub-catchment is in line with the land slope (the sine of slope angle). This suggests dominance of the component term $S^a b$ in Eqn 13. When slope increases from upperslope to midslope, net soil losses also increase. However, as the slope typically decreases from just below the midslope to the bottomslope, the expected net soil loss also decreases until net accumulation is expected to occur at the bottom-end of the transect. The exception in this general trend is for transect 9N, and this occurs because the land slope of this transect always increases right up to the bottom end of the transect, so that WEF is never negative, and no soil accumulation is expected.

Cultivation erosion factor

The predicted spatial variation in net soil redistribution by cultivation is available from the cultivation erosion factor (Eqn 4), calculated using the sub-catchment microtopographical data. The trend of the CEF with the downslope distances for the 6 transects is presented in Fig. 4 (labelled CEF). The CEF generally decreases from a higher positive value at the upperslope to zero about midslope before falling to negative values at the bottomslope. This implies that expected net soil loss due to rotary cultivation would generally occur from the upperslope to the midslope of the sub-catchment. However, net soil accumulation due to cultivation would be expected to take place at the bottom or lower part of the sub-catchment where the cultivation factor is negative.

The general trend described in the prior paragraph occurs for all transects except for transect 9N where the slope increases rapidly through the upperslope, and then much more slowly and continuously down both the midslope and bottomslope (see Fig. 2). This provides a different spatial trend in CEF compared with other transects, with net soil loss by cultivation predicted at almost all parts of the transect, as shown in Fig. 4.

It follows from Eqn 4 that no net soil redistribution is generated by rotary cultivation in any part of the transect with uniform slope. When the slope is uniform, dSt/dx, $d^2(fx)/dx^2$, and so CEF are all zero. This is illustrated clearly for calculation sites in transects 1N, 5N, and 9N and transect 5S (see Figs 2 and 4). This result follows since if the slope is uniform, then the distance of soil redistribution by the rotary hoe is also uniform, and the distance MN in Fig. 1 is zero. Thus no net erosion or accumulation of soil is predicted to occur.

Hence, soil predicted as eroded from the convex upperslope by cultivation is transferred to the concave bottomslope, with limited net gain or loss in the midslope region.

Relative importance of rotary cultivation and overland flow on soil erosion

In addition to the water and cultivation erosion factors (labelled WEF and CEF respectively), Fig. 4 also shows the spatial features of net soil loss based on caesium-137 results (labelled CS). Comparing these spatial trends would be expected to allow assessment of the relative importance of rotary hoe cultivation and overland flow on soil erosion, although Fig. 4 shows the contrast in these 2 trends is not great. With the reservations described in the model section, the independent experimental information provided by caesium-137 deficit is regarded as providing the surest evidence of net soil loss. Hence a similarity in spatial form of variation in either erosion factor or the form of caesium-137 loss with downslope distance is taken to provide support for the significance of that particular form of erosion, and vice versa. Independent direct measurement of the absolute magnitude of net soil loss or accumulation due to either of the 2 erosion processes considered as a function of position for all transects over the 40 years evaluated using the caesium-137 technique clearly would be virtually impossible to obtain, and is certainly not available to the present study. The methodology employed in comparing the predicted spatial patterns of erosion by either process with the evidence from caesium-137 therefore provides a realistic methodology for assessing their relative significance, though not without its limitations.

This comparison with caesium results will be made first for the upper, uniformly convex, parts of the opposing slopes, and then for the lower half of the sub-catchment adjacent to the drainage flow line which divides the sub-catchment into its north and south facing slopes.

Erosion process evaluation in the upper half of the sub-catchment

In general, the caesium-137 (CS) data in Fig. 4 show that the rates of net soil loss from upper, convex slopes are relatively high compared with other parts of the sub-catchment, except for transects 1S and 9S. This general result indicates the more strongly eroded nature of the upperslope of the sub-catchment.

Because of the methodology employed it is valid only to compare the *sign* and the *trend*, rather than the absolute values of the experimentally based (CS) and theoretically derived (CEF, WEF) net soil erosion estimates in Fig. 4. Whilst there are minor differences in trend between the calculated water and cultivation erosion factors (WEF and CEF) shown in Fig. 4, their general trends are very similar except perhaps for transect 1N. Sign and location of sign change are very similar for both WEF and CEF (Fig. 4).

Whilst the calculation of WEF and CEF involves use of sub-catchment microtopographic data in different ways, there are some similarities in the theory for either erosional process. In particular, there is some similarity between the term $d^2f(x)/dx^2$ in the CEF (Eqn 4) and the term dS/ds in the WEF (Eqn 13). However, as noted earlier, there is some indication that the other term, S*b, in Eqn 13 is dominant. Thus the general similarity in form between WEF and CEF in Fig. 4 is somewhat unexpected. This similarity increases the difficulty in distinguishing between these 2 quite different causes of soil displacement. However, the agreement in sign between WEF, CEF, and CS in the upper half of the sub-catchment could indicate that both erosion processes have contributed to the indicated net soil loss. Comparison of the trends in WEF and CEF with the trend in CS is limited because only two caesium samples were taken in the upper half of each transect (Fig. 4). Comparison is also limited by the possibility that the level of net erosion indicated by caesium-137 analysis for the top or upper sample could be an overestimate because of the start-up dilution effect described in the model section. Indeed if this overestimate was real, there could be better general agreement in trend between the caesium-based and the 2 theory-based estimates of net soil loss in the upper half of the sub-catchment.

Only one transect, namely 1N, has any substantial difference in trend between WEF and CEF (Fig. 4); and in this case the trend in neither of these factors agrees with the trend in CS. In the 5 other cases where the trends of the WEF and CEF are similar, the result of trend comparison with CS varies. There is some agreement in the trends of WEF and CEF with CS for transects 1S and 5N, and uncertainty or disagreement in the trend comparison for transects 5S and 9S, and 1N and 9N (Fig. 4).

In summary, for this upper half of the catchment, comparison of the spatial trend in the 3 factors does not show agreement in all cases. This may indicate an inadequate number of caesium samples, that erosion due to causes other than overland flow and cultivation can be significant, that the start-up dilution effect is significant, or that the theory given for these processes is inadequate. Furthermore, because of the general similarity in form of results for CEF and WEF, there is no clear evidence supporting the dominance of either cultivation or flow-driven processes as the major source of net erosion shown by the CS data to have occurred.

Erosion process evaluation in the lower half of the sub-catchment

The caesium-137 results in Fig. 4 show that net soil loss increases in going from midslope to bottomslope for transects 5S, 9S, and 9N; net soil loss decreases for transects 1S and 5N, and alters little in 1N. In the case of transects 5S, 9S, and 1N, neither the overland flow nor the rotary cultivation model predictions support this increasing trend of net soil loss from midslope to bottomslope indicated by CS. For this same lower half of transects, both the water and cultivation erosion factors support the decrease in soil loss indicated by CS for transects 1S and 5N, but differ by indicating net deposition. More directly, both water and cultivation erosion factors support the increase in soil loss shown by CS for 9N. The small decrease in soil loss indicated by CS for the lower half of transect 5N is possibly supported by both the cultivation and water erosion factors.

Summarising results, in 3 of the 6 transects CS values indicate a substantial increase in soil loss in progressing from midslope to bottomslope of the sub-catchment. For these same 3 cases, except for understandable slope reasons in transect 9N, the trends of both WEF and CEF are in the opposite direction to trends in CS, predicting soil accumulation at this segment of the landscape. Thus it seems that there were factors other than erosion by hillside overland flow and rotary cultivation causing soil erosion at this bottom part of the sub-catchment, or else deposition of unlabelled sediment occurred. As mentioned earlier, periodic flooding of the valley bottom and its overflowing intermittent water course, processes not considered in the hillside overland flow or rotary cultivation models, are possible explanations. Such valley flooding was observed to occur during the experimental period, and a number of such events could be expected during the 40-year period involved in the caesium-137 methodology. Another possible explanation given earlier in discussion of caesium-137 results was that β was elevated by exfiltration. This was explored by assuming β to increase from 1 to 1.5, which, however, did not alter the general predicted spatial trend in the water erosion factor.

Conclusions

The relative importance of rotary hoe cultivation and overland flow on soil erosion in this sub-catchment is investigated by comparing the spatial trends in soil loss predicted by theory of these processes with that estimated by the experimental caesium-137 methodology. For the upper convex parts of the opposing slopes of the sub-catchment there were transects where the predicted trends of erosion by cultivation and by overland flow agreed with the trends indicated by caesium-137 measurements, and other transects where this was not the case. As a consequence, for the upper slopes analysis has not clearly supported a dominance of erosion by either cultivation or overland flow.

In the lower half of the transects, except for the transect 9N, both the water and cultivation erosion factors predict net soil accumulation. This prediction is contrary to all the results based on caesium-137 measurement, which indicate no net soil accumulation, but rather, soil loss for all parts of the sub-catchment, even in the bottomslope. As well as this common difference in the sign of soil loss, even the trends expected in net soil loss due to either cultivation or overland water flow commonly disagreed with the trends based on caesium-137 measurement. These discrepancies indicated that the net soil loss indicated by caesium-137 at the bottomslope must be due to factors other than overland hillslope flow or the analysed effect of cultivation. The observed periodic erosive flooding of the water course in the bottomslope of the sub-catchment is one likely cause of this disagreement in trends between the net soil loss based on caesium-137 analysis and model predictions of the 2 erosion processes considered.

Further study would be worthwhile in order to test the expected cause of discrepancy between caesium-137 results and the estimation of erosion models which are most noticeable in the bottom or lower parts of the sub-catchment.

Acknowledgments

The authors wish to express their gratitude to Mr Cyril Ciesiolka of Queensland Department of Natural Resources who provided the topographic data for the study area, and Dr P. Hairsine of CSIRO Land and Water who provided valuable criticism of the manuscript. The authors also acknowledge the assistance received from Mr Aubrey L. Chandica, a cartographer at Faculty of Environmental Sciences, Griffith University.

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Manuscript received 9 February 1999, accepted 4 July 2000

Appendix 1. Notation

The following symbols are used in this paper:

BL : distance of soil being removed on land of uniform slope (m)
BN : distance of soil being removed on land of variable slope (m)
BZ : depth of cultivation of using rotary hoe (m)
c : sediment concentration (kg/m^3)
c_t : sediment concentration at the transport limit (kg/m^3)
D : depth of flow (m)
k : a constant of erosion factor
K : an erodibility factor
K_1 : a constant or factor believed to depend on soil type, runoff rate and Manning's n
MN : effect of rotary cultivation on net soil redistribution on land of variable slope (m)
n : surface roughness coefficient Manning's n (m$^{-1/3}$ s)
q : volumetric water flux (m^3/m.s)
q_s : sediment flux (kg/m.s)
Q : run off rate per unit area (mm/s)
r_1 : radius of rotary hoe (r) minus one half of the cultivation depth BZ (m)
r_2 : radius of rotary hoe (r) minus the cultivation depth BZ (m)
R_c : radius of curvature of the land surface (m)
s : downslope distance (m)
S : land slope (sine of land slope angle)
St : land slope (tangent of land slope angle)
t : time (s)
v : speed of the rotary hoe's general motion (m/s)
V : velocity of water flow (m/s)
x : distance on a horizontal cartesian co-ordinate (m)
θ : angle of land curvature between two points (degree)
ψ : land slope angle (degree)
ϕ : angle between the land slope at B and the chord linking points B-N (degree)
β : soil erodibility parameter

Appendix 2. Derivation of expression for the distance MN in Fig. 1

Figure A1 is an enlargement of the segment ΔBGI of Fig. 1. Land slope at B (St) is given by:

$$St = \tan\psi \text{ or } \psi = \tan^{-1} St \quad [A1]$$

Using curvature theory, the downslope distance (ds) between the points B and J is defined as:

$$\delta s = R_c\, \delta\theta \quad [A2]$$

where R_c is the radius of curvature of the land surface, and $\delta\theta$ is the angular rotation of the radius.

From BGI of Fig. A1, it may be noted that as $\delta\theta \to 0$, then BI → δs and $\delta x \to dx$. Thus, in the limit $\delta\theta$ tends as follows:

$$\delta\theta \to \frac{dx}{R_c \cos\psi} \quad [A3]$$

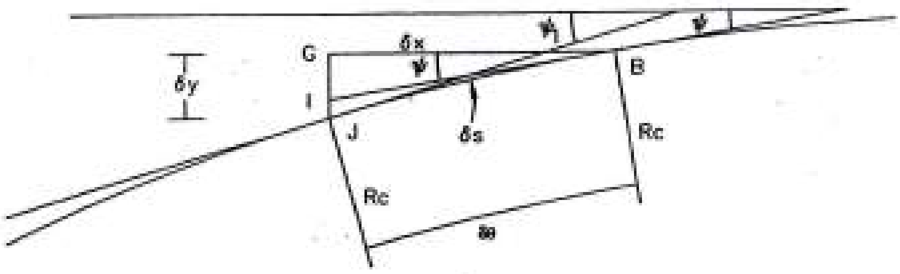

Fig. A1. Curvature of plane element at land slopes of ψ and ψ_1 at points B and J, respectively.

Let $\delta\psi = \psi_1 - \psi$. According to the curvature theorem, $\delta\psi$ is equal to $\delta\theta$ (Chirgwin and Plumpton 1970), therefore R_c in Eqn A3 becomes:

$$R_c = \frac{dx}{\cos\psi\ d\psi} = \frac{dx}{\cos\psi\ d(\tan^{-1} St)}$$

Using a theorem of differential trigonometry in substituting $d(\tan^{-1} St)$, Rc becomes:

$$R_c = \frac{dx\,(1+St^2)}{\cos\psi\ dSt}$$

Moreover, from trigonometry:

$$\cos\psi = \frac{1}{(1+\tan^2\psi)^{0.5}} = \frac{1}{(1+St^2)^{0.5}}$$

so that the value of Rc can be expressed as:

$$R_c = \frac{(1+St^2)^{1.5}}{(dSt/dx)} \qquad [A4]$$

where (dSt/dx) is the rate of slope change with the change dx of horizontal distance x.

Let ϕ be the angle between the land slope at B and the chord linking points B and N (see Fig. A2). Using curvature theory and Eqn A3, the angle of ϕ can be expressed as:

$$\phi = \frac{\delta\theta}{2} = \frac{dx}{2R_c\cos\psi} \qquad [A5]$$

From the Δ BLV, as $\delta\theta \rightarrow 0$ then LV → LM. The distance of LM can be expressed as:

$$LM = BL\ \tan\phi = BL\ \tan\left(\frac{\delta\theta}{2}\right)$$

When $\delta\theta$ is a very small angle, the distance LM becomes:

$$LM = BL\frac{\delta\theta}{2} \qquad [A6]$$

Using ΔLMU (Fig. A2), as $\delta\theta \rightarrow 0$ then MU → MN. Therefore, in limit, the distance of MN can be estimated as:

$$MN = LM\ \tan\psi \qquad \text{or}$$

$$MN = BL\frac{\delta\theta}{2}\tan\psi \qquad [A7]$$

Substituting the Eqn A5 into the Eqn A7, the distance of MN becomes:

$$MN = BL\left\{\frac{dx}{2R_c\cos\psi}\right\}\tan\psi \qquad [A8]$$

An expression for BL in Fig. 1 is now derived. A general feature of the geometry defined in Fig. 1 is the distance downslope moved in one rotation of the hoe by a soil element initially at the point A. This distance is BL. An expression for the distance of BQ can be derived from the ΔBCQ in Fig. 1, as follows. Let AC = r_1 and BC = r_2, and ψ be the angle of the land slope at B. Then:

$$BQ = r_1\ \sin\psi \qquad \text{and then}$$

$$HB = r_1\ (1 + \sin\psi) \qquad [A9]$$

Note that r_1 = CZ – AZ, where CZ is the radius of rotary hoe, and AZ is one half of the cultivation depth, BZ.

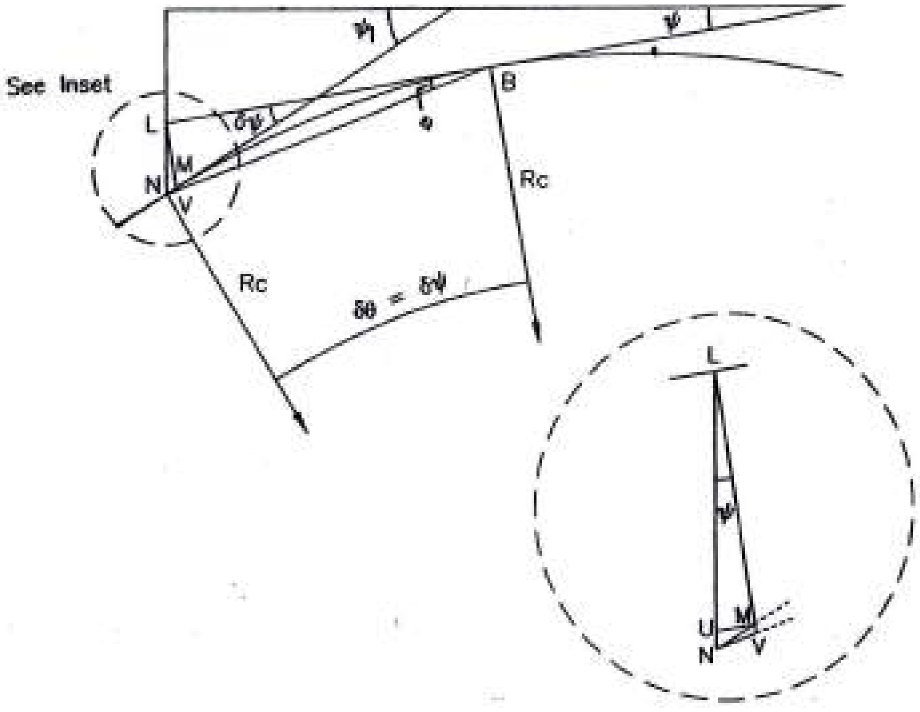

Fig. A2. A triangle showing the distance between two points of M and N.

Using Δ BHK, the distance of BK can be calculated as follows:

$$BK = \frac{r_1(1+\sin\psi)}{\cos\psi} \qquad [A10]$$

When the tractor with rotary hoe moves with a speed v, then in an element of time (dt) the distance of movement is CD, where

$$CD = v\,\delta t$$

In order to relate the movement of the rotary hoe to the land slope, Δ CDT can be used as follows:

$$TC = CD\cos\psi = v\,\delta t\cos\psi \qquad [A11]$$

Now RK = FS = FD – SD. Because SD = ET, and FD = r_1 = EC, then

$$RK = TC = v\,\delta t\cos\psi \qquad [A12]$$

Using Δ RKL (Fig. 1), the distance of KL can be calculated from

$$KL = \frac{RK}{\cos\psi} = v\,\delta t \qquad [A13]$$

Now, BL = BK + KL. Combining Eqns A10 and A13, the distance of BL can be expressed as:

$$BL = \frac{r_1(1 + \sin\psi)}{\cos\psi} + v\,\delta t \qquad [A14]$$

Using Eqn A4 for R_c, and Eqn A14 for distance of BL in Eqn A8, the distance of MN can be expressed as:

$$MN = \frac{\left\{ \frac{r_1(1 + \sin\psi)}{\cos\psi} + v\delta t \right\}\tan\psi}{2\cos\psi} \times \frac{\delta St}{(1 + St^2)^{1.5}} \qquad [A15]$$